Q. How can you tell if the series $\sum_{n=0}^{\infty} a_n$ converges or diverges?

Ex 1 Does $\displaystyle\sum_{k=2}^{\infty} \frac{1}{2k-1}$ converge or diverge?

(a) Converges

(b) Diverges

Ex 2 Does $\displaystyle\sum_{n=1}^{\infty} \frac{1}{ne^n}$ converge or diverge?

(a) Converges

(b) Diverges

Ex 3 Does $\sum_{n=1}^{\infty} -\frac{1}{n}$ converge or diverge?

(a) Converges

(b) Diverges

Note:

Moral:

Thm: (The Direct Comparison Test)

(a) Suppose there is an index n_0 such that _____ for $n \geqslant n_0$. If $\sum_{n=1}^{\infty} b_n$ _____, then $\sum_{n=1}^{\infty} a_n$ _____.

(b) Suppose there is an index n_0 such that _____ for $n \geqslant n_0$. If $\sum_{n=1}^{\infty} b_n$ _____, then $\sum_{n=1}^{\infty} a_n$ _____.

Ex 4 Decide whether $\sum_{n=1}^{\infty} \frac{1}{n^5 + \sqrt{n}}$ converges or diverges.

(a) Converges

(b) Diverges

Q. Could we use a different comparison?

Ex 5 Decide whether $\sum_{n=1}^{\infty} \frac{1}{\ln(n)+4}$ converges or diverges.

(a) Converges

(b) Diverges

Q. Could we use a different comparison?

Ex 6 What about $\sum_{n=1}^{\infty} \frac{5n}{5n^3 - 2}$?

(a) Converges

(b) Diverges

Thm: (Limit Comparison Test) Suppose $a_n \geqslant 0$ and $b_n \geqslant 0$ for all $n \geqslant 1$, and suppose that

_____ (where L is a stand-in for either a real number or ∞).

(a) If _____, then $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ have _____ behavior. (i.e. _____)

(b) If _____ and $\sum_{n=1}^{\infty} b_n$ _____, then $\sum_{n=1}^{\infty} a_n$ _____.

(c) If _____ and $\sum_{n=1}^{\infty} b_n$ _____, then $\sum_{n=1}^{\infty} a_n$ _____

Ex 7 Using $a_n = \dfrac{5n}{5n^3 - 2}$ and $b_n = \dfrac{1}{n^2}$ for all $n \geqslant 1$, decide whether $\sum_{n=1}^{\infty} \dfrac{5n}{5n^3 - 2}$ converges or diverges.

Ex 8 Decide whether $\sum_{n=1}^{\infty} \dfrac{8^n}{3 + 4^n}$ converges or diverges.

(a) Converges

(b) Diverges

Ex 9 Decide (by any means) whether the series $\sum_{n=1}^{\infty} \frac{n^4+1}{e^n}$ converges or not.

(a) Converges

(b) Diverges